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Locus Model for the Pellet's Shiver of Smoke Projectile

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Abstract: The locus equation for the pellet's shiver of smoke projectile was established by using the air friction and the gravitation, and the solving process of equation was presented. The exploding process of smoke projectile was shot by a high speed camera. Test data were used to obtain related coefficients, and the mathematical model was modified further. The results illustrate that air friction effect on the flying locus of shiver is significant.

Key words: military chemistry; pyrotechnics; smokescreen; pellet's shiver; locus equation; mathematical model

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1 Introduction

Smokescreen is an important weapon in optics-electricity countermeasure field due to its cheapness and effectiveness of evading an attack. This promotes advancement of smoke material. Not only can smokescreen shelter visible light, but also infrared[1], even millimeter wave. However, sometimes the high-quality smoke material can not be adopted without reasonable design for the charge of smoke projectile. The purpose of design for the charge is to ensure uniformities of dispersed and integral shapes of smoke material. Under the condition that the physical dimension is relatively fixed, not only has smoke material itself influence on the performance of smoke projectile, but also the radius of pellet's shiver of smoke projectile and the rate of smoke formed. They are key parameters to form smokescreen.

2 The flying locus equation for the shiver of smoke projectile

After smoke projectile blasting in air, the pellet's shivers of smoke agent begin to be accelerated. But the acceleration time is very short compared with the forming process of smoke and the range of action. Therefore, the initial conditions may be assumed as $t=0, v=v_0, R=0$. The pellet's shivers are driven mainly by two forces[2,3]: the gravitation and the air friction. Now assuming the separation angle between the directions of pellet's shiver and horizon is α, the initial velocity of the pellet's shiver is v_0, the mass is m, and then, the velocity of the pellet's shiver is: $v_0 \sin\alpha \geqslant 0$. For $v_0 \sin\alpha \geqslant 0$ (The pellet's shiver flies leanly up and the component of force with respect to the vertical direction of ground is upward).

$$m\frac{\mathrm{d}^2R}{\mathrm{d}t^2} = -D\left(\frac{\mathrm{d}R}{\mathrm{d}t}\right)^2 - mg \tag{1}$$

Translating into:

$$\frac{\mathrm{d}^2R}{\mathrm{d}t^2} = -\frac{D}{m}\left(\frac{\mathrm{d}R}{\mathrm{d}t}\right)^2 - g = -\frac{D}{m}v^2 - g \tag{2}$$

When $v_0 \sin\alpha < 0$,

$$\frac{\mathrm{d}^2R}{\mathrm{d}t^2} = \frac{D}{m}v^2 - g \tag{3}$$

Where R, the displacement of pellet's shiver (the center of circle is the blasting point); g, the gravitational acceleration; D, the friction coefficient of air.

For the sake of simplicity, let $\frac{D}{m} = k^2$.

Equations (2) and (3) with respect to the horizontal direction may be translated into:

$$\frac{\mathrm{d}v_h}{\mathrm{d}t} = -k^2 v_h^2 \tag{4}$$

Equation (2) with respect to the vertical direction may be translated into:

$$\frac{\mathrm{d}v_h}{\mathrm{d}t} = -k^2 v_v^2 - g \tag{5}$$

Equation (3) with respect to the vertical direction may be translated into:

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$$\frac{dv_v}{dt} = k^2 v_v^2 - g \tag{6}$$

The general solution is solved after Equations (4), (5) and (6) are integrated, respectively. Mark $t=0$, then the horizontal velocity of pellet's shiver after blasting is

$$\frac{1}{v_h} = k^2 t + C_1 \Rightarrow v_h = \frac{1}{k^2 t + C_1} \tag{7}$$

Where $C_1 = \frac{1}{v_0 \cos\alpha}$

When $v_0 \sin\alpha > 0$, and $t < \frac{C_2}{k\sqrt{g}}$

$$v_v = \frac{\sqrt{g}}{k}\mathrm{tg}(-k\sqrt{g}t + C_2) \tag{8}$$

$C_2 = \mathrm{arctg}(\frac{k}{\sqrt{g}}v_0 \sin\alpha)$, when $t \geqslant t_c = \frac{C_2}{k\sqrt{g}}$, where t_c is time when the component of velocity with respect to the vertical direction is zero,

$$v_v = \frac{\frac{\sqrt{g}}{k}(1 - e^{2k\sqrt{g}(t-t_c)})}{1 + e^{2k\sqrt{g}(t-t_c)}} \tag{9}$$

When $v_0 \sin\alpha \leqslant 0$

$$v_v = \frac{\frac{\sqrt{g}}{k}(1 + C_3 e^{2k\sqrt{g}t})}{1 - C_3 e^{2k\sqrt{g}t}} \tag{10}$$

Where $C_3 = (v_0 \sin\alpha - \frac{\sqrt{g}}{k})/(v_0 \sin\alpha + \frac{\sqrt{g}}{k})$. Then, the velocity of pellet's shiver at any time can be calculated combined with Equations (7), (8), (9) and (10). The relation between displacement in the horizontal direction x and time t is as follows:

$$x = \frac{1}{k^2}\ln(\frac{k^2}{C_1}t + 1) \tag{11}$$

Combining Equations (8), (9) and (10), the relation between displacement in the vertical direction y and time t is:

$$y = \int_0^t v_v dt \tag{12}$$

The integral value y can be calculated by sum formula while the coordinate position of pellet's shiver at arbitrary time can be determined when the blasting point is assumed as origin of the coordinate.

3 Experimental and discussion

When the obscuring time of a smokescreen is required over 30 s, the combustion rate is usually slow to form desired shapes for applications. In this case, the main combusting process of smoke pellets is under environmental pressure.

The whole process of forming smokescreen is shot by a high speed camera in the test from igniting to falling to the ground, as shown in Figure 1. Based on the center of circle is the blasting point, the pellet's shiver that can be easily distinguished is defined as one reference object, and its geometric locus plane would be vertical with line-of-sight of measurement. Because of the high shot-speed of the camera, the change rate of displacement of the reference object upon photograph may be regarded as the instantaneous velocity. Equation (7) is the easiest way to solve k among Equations (7)、(8)、(9) and (10). k^2 value may be solved by only applying two groups of data (v_1, t_1) and (v_2, t_2).

$$k^2 = (v_{h2} - v_{h1})/v_{h1}v_{h2}(t_1 - t_2) \tag{13}$$

The displacement of pellet's shiver in the horizontal direction with time is shown in Figure 2 by testing.

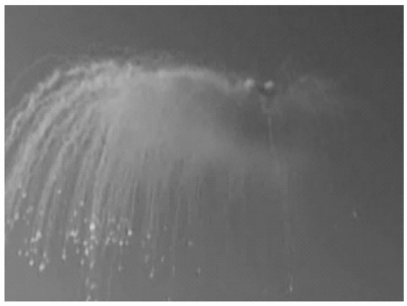

Fig. 1 Wink figure of explosion forming smoke

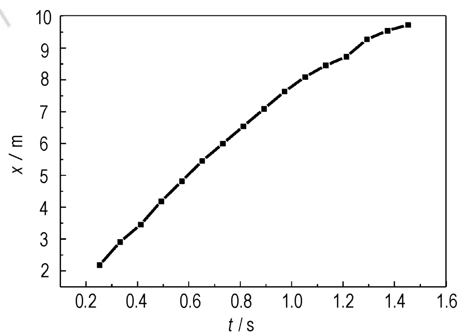

Fig. 2 Relation between displacement in the horizontal direction (x) and time (t)

The trend of velocity degrades rapidly, as shown in Figure 3. The velocity of pellet's shiver varies from 9 m/s to 3 m/s within 1 s. The pellet's shiver is only driven by the air friction in the horizontal direction, so the friction can not be neglected. Because the reference object of pellet's shiver selected is a big-mass solid piece, the rising force of air may be neglected. k^2 is temporarily assumed as a constant, the value k^2 may be derived by curve fitting method combined with Equation (7), $k^2 = 0.07$, and $C_1 = 0.1$. The curve is line B in Figure 3.

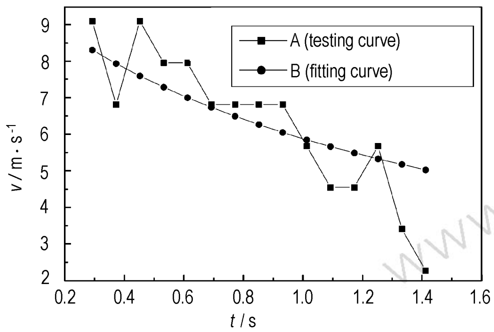

Fig. 3　Relation between rate in the horizontal direction (v) and time (t)

The trend of velocity curve that corresponds to Figure 2 is curve A in Figure 3.

The smoke projectile is mainly made up of smoke pellets, which become into pellet's shivers after blasting, and give off smoke continuously, at the same time, some grain fall off for air friction, then the mass of pellet's shiver become less and less. So $D/m = k^2$, k^2 is a variable during flying. Based on the data of Figure 3, the process that k^2 varies with time is shown in Figure 4.

The value k^2 is unsteady in main process of smoke forming, as shown in Figure 4. Because the pellet's shiver combusts under environment pressure, according to combustion model of Summerfield compound powder[4], $r = C \cdot p$, where r is the combustion rate, C is a constant, p is the environment pressure, the combustion rate of mass is a linear model. The mass of pellet's shiver is descending, however, k^2 may be a constant when density of smoke pellets is very large and combustion rate is very low. The k^2 may be translated into increasing variable with time when the quality of giving off smoke is good and the volume of pellet's shiver varies slightly, then

$$k^2 = \frac{D}{m - at} \tag{14}$$

Where initial mass of pellet's shiver is m, a is coefficient of forming smoke rate. Equation (14) is substituted into Equation (4), and then the curve equation of velocity varying on horizon is

$$v_h = \frac{1}{C_1 - \frac{D}{a}\ln\left(\frac{m}{a} - t\right)} \tag{15}$$

In which $C_1 = \frac{1}{v_0 \cos\alpha} + \frac{D}{a}\ln\frac{m}{a}$, where $0 \leqslant t \leqslant t_e$, t_e is the reaction end time.

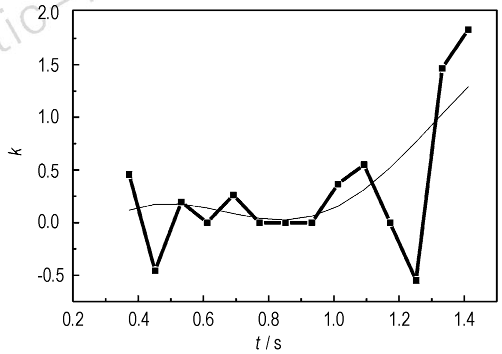

Fig. 4　Curve of the change direction of air friction coefficient (k) versus time (t)

When Equation (14) is substituted into Equation (5) and (6), the variables can not be separated. The solution similar to Equation (15) can not be obtained, but Equations (8), (9) and (10) can be modified by using Equation (14) at any time.

The velocity of pellet's shiver can be described by Equations (7), (8), (9) and (10) when combustion rate of pyrotechnics pellet is low and its molding is under high pressure. The friction coefficient of air D depends on the shape and volume of pellet's shiver, thus it works with precast shiver parameter, such as groove for passes fire and thickness. If groove for passes fire is designed, mass and shape of pellet's shiver may be controlled after blasting, so the value D can be given by calculation or test.

4　Locus model for pellet's shiver

nitial velocity of pellet's shiver is 15 m/s by testing. Based on the model and parameters obtained in the above section, when $v_0 \sin\alpha > 0$, namely $0 < \alpha \leqslant \frac{\pi}{2}$, critical time t_c of equation replacing with angle α is (for $t_c = \mathrm{arctg}(\frac{k}{\sqrt{g}} v_0 \sin\alpha) / k\sqrt{g}$).

Based on nine angles: $-\frac{\pi}{2}, -\frac{3}{8}\pi, -\frac{\pi}{4}, -\frac{\pi}{8}, 0, \frac{\pi}{8}, \frac{\pi}{4}, \frac{3}{8}\pi, \frac{\pi}{2}$, the locus of pellet's shiver is investigated.

The critical times t_c of equation replacing that correspond to the latter four angles are 0.54 s, 0.88 s, 1.04 s, 1.08 s, respectively. At the same time, two points at 2 s and 3 s are enforced. Substituting these data into Equations (11) and (12), individual coordinate position may be obtained. Figures 6 and 7 are locus of pellet's shiver on two-dimensional plane in 3s after blasting, the origin of coordinate is the blasting point. Those curves in Figure 6 are simulating locus for pellet's shiver based on Equations (11) and (12) while those curves in Figure 7 are under no air friction.

Each curve in figures represents one flying locus of shiver. Drop height is up to 50 m by comparing Figure 6 with Figure 7. Obviously, the air friction plays an important role in the forming process of smokescreen.

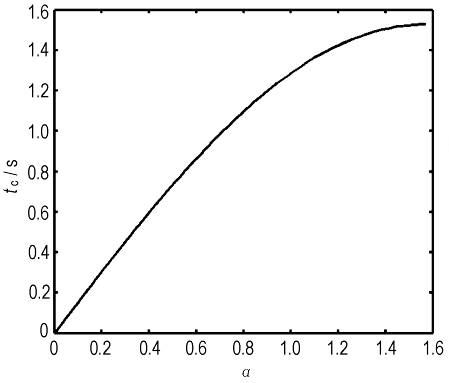

Fig. 5 Relation between separation angle (α) and critical time (t_c)

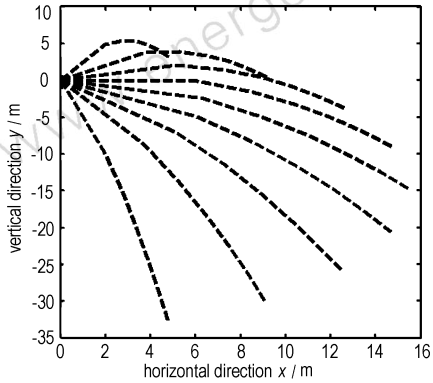

Fig. 6 Locus simulating for a pellet's shiver in air

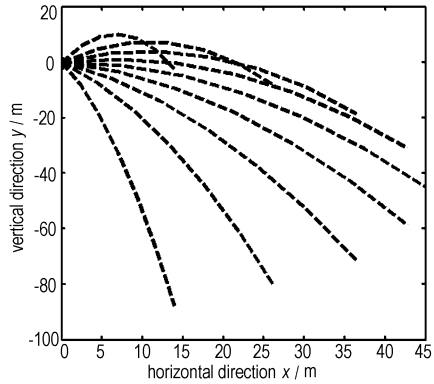

Fig. 7 Locus simulating without air

5 Conclusions

By defining the reference object based on large distinguished pellet's shiver and neglecting effect of the rising force on shiver, equation for the pellet's shiver of smoke projectile is established and the flying locus of shiver is further derived by only considering the air friction and the gravitation. Experimental results demonstrate that the air friction is one factor that can not be neglected. The mathematical model is established by taking the air friction and the gravitation into account and the related parameters are obtained. Therefore, the model can be used to describe flying locus for shiver of smoke projectile.

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烟幕弹的发烟剂碎块轨迹模型

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摘要：利用飞散物在运动过程中所受到的空气阻力和重力建立飞散物运动方程,用高速摄影仪拍摄了烟幕爆炸成烟过程,并用实验结果求出了相关系数,校正了模型。结果发现空气阻力对烟幕成型有明显的影响。

关键词：军事化学；烟火技术；烟幕；发烟剂碎块；轨迹方程；数学模型

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